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Unification with low string scale

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Abstract: I argue that in open-string theory with hierarchically small (or large) extra dimensions, gauge groups can unify naturally with logarithmically-running coupling constants at the high Kaluza-Klein (or string-winding) scale. This opens up the possibility of rescuing the standard logarithmic unification at $M_U \sim 10^{15-18}$ GeV even if the fundamental-string scale is much lower, at intermediate or possibly even electroweak scales. The argument, however, also shows why a low type-I string scale may not suffice to obliterate the ultraviolet problems associated with the gauge hierarchy.

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The perturbative unification of the standard-model gauge coupling constants extrapolated to $M_U \sim 10^{15-18}$ GeV , is one of the few solid hints we have about physics beyond the standard model. The weakly-coupled heterotic string, compactified near the string scale $M_s^{het} \sim 10^{17-18}$ GeV, is naturally compatible with such a unification scenario [1, 2] – the one-to-two order-of-magnitude discrepancy between M_U and M_s^{het} being a relatively minor nuisance, when compared to the logarithmic distance separating these scales from the electroweak scale [3]. More recently there has been renewed interest in the possibility of departing from this traditional point of view, and allowing hierarchically-different string and compactification scales [4, 5, 6, 7, 8, 9, 10]. It has been argued in particular that the string scale could be lowered in a controlled way, either in type-I theory where this is allowed by the tree-level relations [4, 5, 9, 10], or in the heterotic theory, where the tree-level relation ($M_s^{het} \sim g_{\rm YM} M_{\rm Planck}$) seems to forbid it, but one may take hierarchically-small bare Yang-Mills couplings and then invoke large-radius threshold effects to drive some of them to their observed low-energy values [7]. The fact that such exotic scenaria are not immediately ruled out by experiment and might be even testable in the near future [11], is certainly sufficient motivation by itself to continue investigating them further.

These scenarios share with the more conservative point of view the usual problem of vacuum stability after supersymmetry breaking. They seem to face furthermore some extra difficulties : if one abandons, in particular, the conventional unification at high M_U by logarithmic running, one gives up a 'natural' explanation for the value of $\sin^2\theta$, while facing at the same time the risk of totally uncontrollable proton decay. Ideas on how to proceed have been discussed in references [12, 10, 9, 13]. It has been argued in particular that large-radius threshold effects can accelerate unification without affecting the value of $\sin^2\theta$ [12], or that this latter may be fixed by tree-level string relations without group unification [10]. Proton decay, on the other hand, could be conceivably suppressed by discrete gauge symmetries [9], by a weakly-gauged custodial U(1) [13, 10], or by the higher-dimensional nature of the interactions [12].

One assumption often made in these discussions is that the string scale sets the ultraviolet cutoff, and that logarithmic unification can be at best achieved at or below this scale. Here I will try to explain why in (weakly-coupled) type-I theory this naive intuition need not be valid. I will show in particular, using the results of [14], that in the presence of hierarchically small or large extra dimensions, some 4d gauge couplings can run logarithmically and unify at the super-heavy Kaluza-Klein, respectively string-winding scale. This opens up the possibility of rescuing in a modified form the successful unification at $M_U \sim 10^{15-18}$ GeV, even if the type-I string scale (M_s^I) is much lower, in the intermediate or even in the TeV range. More generally, we will see that superheavy ($M \gg M_s^I$) momentum or winding modes can give power-like radiative corrections to 4d gauge couplings – a warning that lowering the type-I string scale may not suffice to obliterate the UV problems, which are usually associated with the existence of the gauge hierarchy.

Let me first explain the main point in the simplest context of a $T^4/Z_2 \times T^2$ type-I compactification, with N=2 supersymmetries in four dimensions [15]. These are the models whose threshold effects have been analyzed in ref. [14]. The background has 32 D9-branes, as well as 32 D5-branes transverse to the T^4/Z_2 orbifold. Let g_s be the string coupling constant, and v_4 and v_2 be, respectively, the volume of the orbifold and the area of the two-torus, both measured in units of $2\pi\sqrt{\alpha'_I}$. Dualizing the two coordinates of the torus gives an equivalent background with D7-branes and D3-branes, and with

$$\tilde{v}_2 = 1/v_2 \qquad \text{and} \qquad \tilde{g}_s = g_s/v_2\,. \tag{1}$$

I will refer to these dual descriptions as type I and type I', respectively, and change freely from one to the other. There are two types of gauge groups – those living on the type I D5-branes or equivalently the type-I' D3-branes, and those living on the type I D9-branes or type-I' D7-branes. The Planck mass and the two tree-level Yang-Mills couplings in four dimensions read

$$g^2_{\rm YM}\Big|_{D3} = v_4\; g^2_{\rm YM}\Big|_{D7} = 2\sqrt{2}\pi\;\tilde{g}_s \qquad \text{and} \qquad M^2_{\rm Planck} = \frac{v_4\tilde{v}_2}{2\pi\alpha'_I\tilde{g}_s^2}\,. \tag{2}$$

We will be interested in the following corner of moduli space : v_4 and $\tilde{g}_s$ of order one, and $\tilde{v}_2 = 1/v_2 \gg 1$ with the two radii of the torus roughly equal. The 4d Planck scale is in this limit hierarchically large, when compared to the fundamental string tension. The type-I Kaluza-Klein excitations, which are dual to type-I' winding states, have masses $M_{\rm KK} \sim \sqrt{\tilde{v}_2/\alpha'_I}$, of order the 4d Planck scale. The key remark for our purposes here is that the logarithmic running of the four-dimensional gauge couplings does not stop at the (much lower) string scale, but continues all the way up to $M_{\rm KK}$.

To understand why notice first that, because of $N=2$ supersymmetry, only short BPS states renormalize the gauge coupling constants. Since all excited states of an open string are non-BPS, the fundamental string scale drops out completely from the loop corrections [16, 14]. The final expression at one-loop is formally identical to that of a six-dimensional gauge field theory, compactified down to four dimensions on a (tiny in type-I language) two-torus, and with the quadratic divergence subtracted out. To be more specific, take the torus to be orthogonal and let R_1 and R_2 be its two radii in the type I picture (so that $v_2 = R_1R_2/\alpha'_I$). The one-loop contribution of a mutliplet in the rth representation of some gauge-group factor reads [14]

$$\Delta\frac{4\pi^2}{g^2_{YM}(\mu)} = \pm C_r\int_0^\infty \frac{dt}{t}\, e^{-(\mu^2+M_r^2)t}\left(\sum_{n_1,n_2\in Z} e^{-\left[(\frac{n_1}{R_1})^2+(\frac{n_2}{R_2})^2\right]t} - \frac{\pi R_1R_2}{t}\right), \tag{3}$$

where C_r is the quadratic Casimir, M_r is the mass of the mutliplet, μ is an infrared cutoff, and the sign is plus for hypermutliplets and minus for the (adjoint) vector. The $1/t$ subtraction terms, transformed to the transverse closed-string channel, sum up over all representations to zero — this is guaranteed by tadpole cancellation as explained in ref. [14]. For inverse radii roughly equal, and much larger than all other mass scales, we may think of the expression in parenthesis as a particular way of imposing an ultraviolet (Pauli-Villars type) cutoff $M_{KK} \sim 1/R_i$ to the standard four-dimensional loop corrections. The gauge-coupling constants will thus run up to this cutoff, much higher than string scale, as advertized.

Two important remarks are here in order . First, if the two radii are hierarchically different, say $R_1 \gg R_2$, then the lighter Kaluza-Klein modes (with masses n_1/R_1) will contribute effectively up to the heavier, $1/R_2$, scale. The theory is in this range five-dimensional and the couplings 'evolve' with a power-law, i.e. receive large (non-logarithmic) threshold corrections. This is the scenario of accelerated unification considered in refs. [12] – but with the role of the UV cutoff played again by the heaviest Kaluza-Klein scale, rather than by the tension of the type-I string. To avoid such large non-logarithmic effects we must insist that the (complex-structure) modulus R_2/R_1 be of order one, i.e. that there are no runaway moduli other than the area of the two torus. The second remark concerns the gauge hierarchy : one may have thought that the fine tuning problems caused by large radiative corrections are obliterated if we lower the string tension to the TeV scale. What we have seen, however, in this toy model is that Planck-scale physics still controls the radiative corrections to 4d gauge couplings. Any statement on the gauge hierarchy needs, therefore, to be further qualified.

A heuristic explanation of our result is as follows: for generic type-I Wilson lines (i.e. for generic separations of the type-I' D-branes on the torus) the mass of the charged gauge bosons is of the order of the Kaluza-Klein scale. It is therefore not surprising that the gauge couplings unify in general up there, and not at the much lower fundamental string scale. Furthermore, the ultraviolet regime of open-string theory is governed by the infrared regime of (super)gravity. Turning on a background gauge-field strength perturbs the equilibrium of the D-branes, which in the type-I' language live in a large transverse two-dimensional space. The leading (F^2) perturbation to vacuum energy exhibits therefore the characteristic two-dimensional logarithmic dependence, and can even grow linearly with radius if the torus becomes effectively one-dimensional at larger scales. In applying the argument to D3 branes one must recall that they couple to twisted massless closed-string modes, which are localized at orbifold fixed points. Thus even if the orbifold is also large, the dominant effect stays linear or logarithmic.

Such heuristic considerations can be perhaps extended to stable $N = 1$ or even $N = 0$ backgrounds. A more precise argument can be given for $N = 1$ orbifold vacua at one-loop order: in type-I' language these vacua are made out of D3-branes, D7-branes transverse to one of the three complex compactified dimensions, and corresponding orientifolds. The one-loop open-string diagrams can be classified according to the amount of supersymmetry they preserve. For instance, a (33) amplitude without an orbifold twist preserves $N = 4$ supersymmetry, while a (37) amplitude without twist or a (33) amplitude with a Z_2 twist preserve $N = 2$ supersymmetry. Now the contributions of $N = 4$ sectors vanish because of helicity-supertrace formulae [17], while those of

$N=1$ sectors do not depend on the compactification moduli. The $N=2$ contributions, on the other hand, are blind to the excited open-string states and will be cut off at high energy by the appropriate Kaluza-Klein scale, as already explained. Thus, if all complex-structure moduli are order one, the gauge couplings should run logarithmically, first with the $N=1$ β-functions up to string scale, and then with modified $N=2$ β-functions up to the much heavier Kaluza-Klein scale. This should be straightfoward to check in some of the explicit $N=1$ type I models [18].

It is also interesting to ask how our discussion would look from the dual heterotic viewpoint. For the $T^4/Z_2\times T^2$ compactification, the dictionnary between various moduli is as follows [19]:

$$S_I = S_{het} \qquad S_I' = T_{het} \qquad \text{and} \qquad U_I = U_{het}\,, \tag{4}$$

where the real parts of the S's determine as usual the 4d Yang Mills couplings (there are two of those on the type I side, corresponding to D9-brane and D5-brane gauge groups), the U's are complex-structure moduli, and T_{het} is the torus-area modulus measured in units of the heterotic string scale. Assuming the type-I gauge couplings to be of order one implies that T_{het} is also order one, i.e. that the two-torus is roughly of heterotic-string size. In the limit of interest ($v_2 \ll 1$) this is hierarchically smaller than the type-I string size, which is in turn comparable to the orbifold volume ($v_4 \sim 1$). To keep S_{het} of order one we must, furthermore, take the heterotic string coupling hierarchically large. We are thus dealing with a very strongly-coupled heterotic string vacuum, compactified on a very large Z_2 orbifold – a situation in which the semiclassical approximation cannot be trusted. Nevertheless, thanks to the extended $N=2$ supersymmetry, the one-loop corrections to gauge couplings are (perturbatively) exact, and in this case are furthermore independent of the orbifold volume [20]. They evolve, therefore, logarithmically up to the heterotic string cutoff, M_s^{het}, which is of the same order as the Kaluza-Klein scale on the two-torus. This agrees of course with what we found on the type-I side.

The fact that one-loop threshold effects can be insensitive to certain moduli was used in ref. [21] in an early effort to construct heterotic vacua that are perturbatively controllable, despite a large compactification scale. As our discussion here shows, this may work for quantities protected by extended supersymmetry, but is in general problematic otherwise. The (light) open-string scale, in particular, should enter as an ultraviolet cutoff in $N=1$ sectors of orbifold compactifications, a large effect that is totally invisible within the heterotic loop expansion.

Let me conclude with some comments on how the unification scenario may apply to a realistic type-I string model. Clearly, in view of the above discussion, the gauge couplings can at best run logarithmically with the MSSM β-functions up to the type-I string scale. They could then continue their logarithmic running up to the heavier unification scale, but with modified β-functions coming from certain $N=2$ sectors of the theory – those sensitive to the large Kaluza-Klein scale. Now the low-energy prediction for $\sin^2\theta$ depends only on the ratio $(b_1 - b_2)/(b_2 - b_3)$, with b_i the one-loop β-function coefficients. This quantity is (numerically) robust under the addition of complete SU(5) representations, or under minimal $N = 2$ extensions of the MSSM particle content [12], see also [22]. It is thus plausible to hope that the successful prediction of $\sin^2\theta$ can be retained despite the β-function modification at M_s^I. Pushing the unification scale up can also suppress the conventional channel of proton decay, through exchange of heavy (X and Y) gauge bosons. Since the type I scale is however low, one would still need to suppress the dangerous channels mediated by exchange of excited open strings.

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